Mathematics

1. Consider 5 independent Bernulli's trails each with probability of sucess p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval

(1) $\left(\frac{11}{12}, 1\right]$ **(2)** $\left(\frac{1}{2}, \frac{3}{4}\right]$

(3) $\left(\frac{3}{4}, \frac{11}{12}\right]$ **(4)** $\left(0, \frac{1}{2}\right]$

Ans: **[4]** P (at least one failure) =1 - P (all success)

$$1 \geq 1 - p^5 \geq \frac{31}{32}$$

$$\Rightarrow \quad 0 \leq p^5 \leq 1 - \frac{31}{32} \leq \frac{1}{32}$$

$$\Rightarrow \quad 0 \leq p^5 \leq \left(\frac{1}{2}\right)^5$$

$$\Rightarrow \quad p \in \left[0, \frac{1}{2}\right]$$

2. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is

(1) 132 **(2)** 144

(3) – 132 **(4)** – 144

Ans: **[4]** Coeff of x^7 in the expansion of $\left((1-x)(1-x^2)\right)^6$

Coeff of x^7 in the expansion of $(1-x)^6 (1-x^2)^6$

Coeff of x^7 in the expansion of

$$\left({}^6C_0 - {}^6C_1x + {}^6C_2x^2 - ... + {}^6C_6x^6\right).\left({}^6C_0 - {}^6C_1x^2 + {}^6C_2x^4 - ... + {}^6C_6x^{12}\right)$$

$$= \left(-{}^6C_0\right)\cdot\left(-{}^6C_5\right) + {}^6C_2\left(-{}^6C_3\right) + \left(-{}^6C_3\right)\left(-{}^6C_1\right)$$

$$= 36 + (-300) + 120$$

$$= -144$$

3. $\lim_{x\to 2}\left(\frac{\sqrt{1 - \cos\{2(x-2)\}}}{x-2}\right)$

(1) equals $\frac{1}{\sqrt{2}}$ **(2)** does not exist

(3) equals $\sqrt{2}$ **(4)** equals $-\sqrt{2}$

Ans: **[2]** $\lim_{x\to 2}\frac{\sqrt{1-\cos 2(x-2)}}{x-2}$

$$= \lim_{x\to 2}\frac{\sqrt{2}\left|\sin(x-2)\right|}{x-2}$$

$$\text{LHL} = = \lim_{x\to 2^-}\frac{\sqrt{2}\left|\sin(x-2)\right|}{x-2} = \lim_{x\to 2^-}\frac{-\sqrt{2}\sin(x-2)}{x-2}$$

$$= -\sqrt{2}$$

$$\text{RHL} = \lim_{x\to 2^+}\frac{\sqrt{2}\left|\sin(x-2)\right|}{x-2} = \lim_{x\to 2^+}\frac{\sqrt{2}\sin(x-2)}{x-2} = \sqrt{2}$$

LHL $\neq$ RHL

The limit does not exist

4. Let R be the set of real numbers

Statement -1

$A = \{(x, y) \in R \times R : y - x$ is an integer$\}$ is an equivalence relation on R.

Statement -2

$B = \{(x, y) \in R \times R : x - \alpha y$ for some rational number α $\}$ is an equivalence relation on *R*.

(1) Statement -1 is false, Statement -2 is true

(2) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for statement-1

(3) Statement-1 is true, Statement -2 is true, Statement-2 is not correct explanation for Statement-1

(4) Statement -1 is true, Statement-2 is false

Ans: **[4]** Statement-1 $(x, y) \in A \Rightarrow (x - x)$ integer

$x - x = 0 \,\forall x \Rightarrow$ A is reflexive

$(y - x)$ integer $\Rightarrow$ $(x - y)$ integer

$\therefore$ A is symmetric

$(y - x)$ integer and $(z - y)$ integer

$\therefore$ $(y - x) + (z - y) = (z - x)$ integer

Hence $xAz \Rightarrow$ A in transitive

$\therefore$ A is equivalance

Statement-2 $(x, y) \in B \Rightarrow$ $x = \alpha y$, for α, some rational

$(0, 1) \in B$ as $0 = 0.1, 0$ is a rational no.

but $(1, 0) \notin B$ as $1 = \alpha . 0$ there is no rational α existing.

Hence B is not symmetric

5. Let α, β be real and z be complex number. If $z^2+\alpha z+\beta=0$ has two distinct roots on the line Re $z=1$, then it is necessary that

(1) $\beta \in (1,\infty)$ **(2)** $\beta \in (0,1)$

(3) $\beta \in (-1,0)$ **(4)** $|\beta|=1$

Ans: **[1]** The coefficients of quadratic equation

$$z^2+\alpha z+\beta=0$$

are real. Therefore complex roots exists in conjugate pair. Let

$$z_1=1+iy,\ z_2=1-iy$$

$$z_1+z_2=-\alpha \Rightarrow \alpha=-2$$

$$\beta=z_1z_2=1+y^2 \in (1,\infty)$$

[For $\beta=1$, roots are equal]

6. $\frac{d^2x}{dy^2}$ equal

(1) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$ **(2)** $\left(\frac{d^2y}{dx^2}\right)^{-1}$

(3) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ **(4)** $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

Ans: **[1]** $\frac{d^2x}{dy^2}=\frac{d}{dy}\left(\frac{dx}{dy}\right)=\frac{d}{dy}\left(\frac{d}{dy/dx}\right)$

$$=\frac{d}{dy}\left(\frac{d}{dy/dx}\right)\frac{dx}{dy}$$

$$=-\frac{1}{\left(\frac{dy}{dx}\right)^2}\cdot\frac{d^2y}{dx^2}\cdot\frac{dx}{dy}$$

$$\frac{d^2x}{dy^2}=-\frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3}$$

7. The number of values of k for which the linear equations

$$4x+ky+2z=0$$

$$kx+4y+z=0$$

$$2x+2y+z=0$$

possess a non-zero solutoin is

(1) zero **(2)** 3

(3) 2 **(4)** 1

Ans: **[3]** For non-zero

$$\begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix}=0$$

$\Rightarrow$ $k^2-6k+8=0$

$\Rightarrow$ $(k-2)(k-4)=0 \Rightarrow k=2,4$

$\therefore$ Number of solution = 2

8. **Statement -1**

The point A (1, 0, 7) is the mirror image of the point B(1, 6, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$

Statement -2

The line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ bisects the line segment joining A (1, 0, 7) and B (1, 6, 3)

(1) Statement -1 is false, Statement -2 is true

(2) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for statement-1

(3) Statement-1 is true, Statement -2 is true, Statement-2 is not correct explanation for Statement-1

(4) Statement -1 is true, Statement-2 is false

Ans: **[3]** mid pt of AB $\equiv (2,3,5)$ lies on the line

dr of AB $\equiv (0,6,-4)$

dr of given $\equiv (1,\ 2,\ 3)$

$0(1)+6(2)+(-4)(3)=0$

$\therefore$ AB is perpendicular to the given line

$\therefore$ B is mirror image of A

Statement (1) is true .

Statmeent (2) is also true But not sufficient.

9. Consider the following statements
P : Suman is brilliant
Q : Suman is rich
R : Suman is honest
The negation of the statment "Suman is brilliant and dishonest if and only if sumna is rich" can be expressed as

(1) $\sim (P \wedge \sim R) \leftrightarrow Q$ **(2)** $\sim P \wedge (Q \leftrightarrow \sim R)$

(3) $\sim (Q \leftrightarrow (P \wedge \sim R))$ **(4)** $\sim Q \leftrightarrow \sim P \wedge R$

Ans: **[3]** Given Statement is equivalent to

$(P \wedge \sim R) \leftrightarrow Q$

Negation is $\sim \left[(P \wedge \sim R) \leftrightarrow Q\right]$

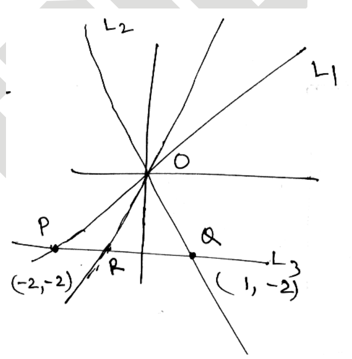
10. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectivley. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement -1
The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$.

Statement -2
In any triangle, bisector an angle divides the trianle into two similar triangles.

(1) Statement -1 is false, Statement -2 is true
(2) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for statement-1
(3) Statement-1 is true, Statement -2 is true, Statement-2 is not correct explanation for Statement-1
(4) Statement -1 is true, Statement-2 is false

Ans: **[4]**



$$\frac{PR}{PQ} = \frac{OP}{OQ}$$

$$= \frac{2\sqrt{2}}{\sqrt{5}}$$

Statement (1) is ture
Statment (2) is False

11. A man saves Rs 200 in each of the first three months of his services. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service wil be Rs. 11040 after

(1) 21 months **(2)** 18 months
(3) 19 months **(4)** 20 months

Ans: **[1]** Let number of months = n

Total saving = 200 + 200 + $\left(200 + 240 + 280 + \ldots \text{ to } (n-2) \text{ terms}\right)$

$\Rightarrow 11040 = 400 + \frac{n-2}{2}\left(2(200) + (n-3)40\right)$

$\Rightarrow 10640 = (n-2)\left(200 + (n-3)20\right)$

$\Rightarrow 532 = (n-2)\,(10 + n - 3)$

$\Rightarrow = (n-2)\,(n+7)$

$\Rightarrow n^2 + 5n - 546 = 0$

$\Rightarrow (n-21)\,(n+26) = 0$

$\Rightarrow n = 21 \quad (\because n > 0)$

12. Equations of the ellipse whose axes are the axes of coordinates and which passes through the point $(-3, 1)$ and has eccentricity $\sqrt{2/5}$ is

(1) $5x^2 + 3y^2 - 32 = 0$ **(2)** $3x^2 + 5y^2 - 32 = 0$

(3) $5x^2 + 3y^2 - 48 = 0$ **(4)** $3x^2 + 5y^2 - 15 = 0$

Ans: **[2]** $e^2 = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{b^2}{a^2} = 1 - e^2 = 1 - \frac{2}{5} = \frac{3}{5}$

$\Rightarrow a^2 : b^2 = 5 : 3$

Equation of ellipse

$\frac{x^2}{5} + \frac{y^2}{3} = k \quad \ldots (1)$

(1) passes through $(-3,\ 1)$

$\therefore k = \frac{(-3)^2}{5} + \frac{(1)^2}{3} = \frac{9}{5} + \frac{1}{3} = \frac{32}{15}$

Equation of ellipse

$\frac{x^2}{5} + \frac{y^2}{3} = \frac{32}{15}$

$\Rightarrow 3x^2 + 5y^2 = 32$

13. If $A = \sin^2 x + \cos^4 x$ then for all real x.

(1) $\frac{3}{4} \le A \le \frac{13}{16}$ **(2)** $\frac{3}{4} \le A \le 1$

(3) $\frac{13}{16} \le A \le 1$ **(4)** $1 \le A \le 2$

Ans: **[2]** $A = \sin^2 x + \cos^4 x$

$= \cos^4 x - \cos^2 x + 1$

$= \left(\cos^2 x - \frac{1}{2}\right)^2 + \frac{3}{2}$

Now $0 \le \left(\cos^2 x - \frac{1}{2}\right)^2 \le \frac{1}{4}$

$\Rightarrow \quad \frac{3}{4} \le A \le 1$

14. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2} dx$ is

(1) $\log 2$ **(2)** $\pi \log 2$

(3) $\frac{\pi}{8} \log 2$ **(4)** $\frac{\pi}{2} \log 2$

Ans: **[2]** $I = \int_0^1 \frac{8\log(1+x)}{1+x^2} \cdot dx$

$x = \tan\theta$

$dx = \sec^2 \theta$

$= \int_0^{\pi/4} \frac{8\log(1+\tan\theta)}{1+\tan^2\theta} .\sec^2\theta\, dx$

also $I = \int_0^{\pi/4} 8\log\left[1+\tan\theta\left(\frac{\pi}{4}-\theta\right)\right].d\theta$

$= \int_0^{\pi/4} 8\log\left[1+\frac{(1-\tan\theta)}{1+\tan\theta}\right]\cdot d\theta$

$= \int_0^{\pi/4} 8\log\left(\frac{2}{1+\tan\theta}\right)\cdot d\theta \; . . . (2)$

using equaiton (1) and (2)

$2I = \int_0^{\pi/4} 8\left[\log(1+\tan\theta)+\log\left(\frac{2}{1+\tan\theta}\right)\right]\cdot d\theta$

$= \int_0^{\pi/4} 8\log 2 \cdot d\theta = 8\log 2\left(\frac{\pi}{4}\right)$

$I = \pi \log 2$

15. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x+2y+3z=4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$

(1) $5/3$ **(2)** $2/3$

(3) $3/2$ **(4)** $2/5$

Ans: **[2]** Let angle between line and plane is θ then

dr's of line : $(1, 2, \lambda)$

dr's of normal plane $(1, 2, 3)$

so, $\cos(90-\theta) = \frac{1\cdot 1 + 2.2 + \lambda\cdot 3}{\sqrt{\lambda^2+4+1}\cdot\sqrt{9+4+1}}$

$\sin\theta = \frac{3\lambda+5}{\sqrt{14}\,\sqrt{\lambda^2+5}}$

given $\cos\theta = \sqrt{\frac{5}{14}} \Rightarrow \sin\theta = \frac{3}{\sqrt{14}}$

Hence $\frac{3\lambda+5}{\sqrt{14}\sqrt{\lambda^2+5}} = \frac{3}{\sqrt{14}}$

$\Rightarrow \quad 9(\lambda^2+5) = (3\lambda+5)^2$

$\Rightarrow \quad 30\lambda = 20$

$\Rightarrow \quad \lambda = \frac{2}{3}$

16. For $x \in \left(0, \frac{5\pi}{2}\right)$, define

$f(x) = \int_0^x \sqrt{t} \quad \sin t\, dt$

Then f has

(1) local maximum at π and local minimum at 2π

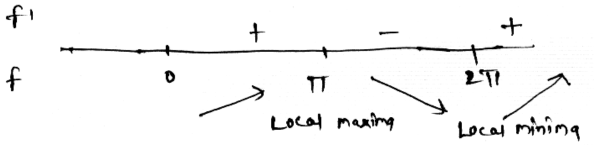
(2) lcoal maximum at π and 2π

(3) lcoal minimum at π and 2π

(4) local minimum at π and local maximum at 2π

Ans: **[1]** $f(x) = \int_0^x \sqrt{t}\sin t\, dt$

applying newton leibnitz formula



$f'(x) = (\sqrt{x} \cdot \sin x) \cdot 1 - 0$

$\Rightarrow \quad f'(x) = \sqrt{x} \cdot \sin x$

Hence $f(x)$ is maximum at $x = \pi$

and local minima at $x = 2\pi$

17. The domain of the function

(1) $f(x) = \dfrac{1}{\sqrt{|x| - x}}$ is **(2)** $(-\infty, \infty) - \{0\}$

(3) $(0, \infty)$ **(4)** $(-\infty, 0)$

Ans: **[4]** $f(x) = \dfrac{1}{\sqrt{|x| - x}}$

Now $|x| - x > 0$

case 1 $\rightarrow$ $x > 0$ $\quad x - x > 0$

$0 > 0 \Rightarrow x = \phi$

case 2 $x < 0$

$-x - x > 0$

$\Rightarrow \quad -2x > 0$

$x < 0 \;\Rightarrow\; x \in R$

Hence $x \in (-\infty, 0)$

18. If the mean deviation about the median of the numbers a, 2a, . . ., 50 a is 50, then $|a|$ equals

(1) 5 **(2)** 2

(3) 3 **(4)** 4

Ans: **[4]** $x_i = a, 2a \ldots 25a, 26a, \ldots 50a$

Median $= \dfrac{25a + 26a}{2} = 25.5a$

Now $(x_i - \bar{x}) = 24.5|a|, 23.5|a| \ldots 0.5|a|, 0.5|a| \ldots 23.5|a|, 24.5|a|$

$\sum |x_i - \bar{x}| = 2(0.5|a| + |a| + 1.5|a| + \ldots + 24.5|a|)$

$= 2 \times \dfrac{25}{2}\left[0.5|a| + 24.5|a|\right]$

$= 625|a|$

Mean deviation $= \dfrac{\sum |x_i - \bar{x}|}{n}$

$= \dfrac{625|a|}{50} = 50$

$\Rightarrow \quad |a| = 4$

19. If $\vec{a} = \dfrac{1}{\sqrt{10}}\left(3\hat{i} + \hat{k}\right)$ and $\vec{b} = \dfrac{1}{7}\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$, then the value of $\left(2\vec{a} - \vec{b}\right) \cdot \left[\left(\vec{a} \times \vec{b}\right)\left(\vec{a} + 2\vec{b}\right)\right]$ is

(1) 3 **(2)** -5

(3) -3 **(4)** 5

Ans: **[2]** $|\vec{a}| = 1, |\vec{b}| = 1, \vec{a}.\vec{b} = 0$

$(2\vec{a} - \vec{b}) \cdot \left[(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})\right]$

$= (2\vec{a} - \vec{b}) \cdot \left[(\vec{a} \times \vec{b}) \times \vec{a} + 2(\vec{a} \times \vec{b}) \times \vec{b}\right]$

$= (2\vec{a} - \vec{b}) \cdot \left[(\vec{a} \cdot \vec{a})\,\vec{b} - (\vec{a} \cdot \vec{b})\,\vec{a} + 2\left\{(\vec{a} \cdot \vec{b})\,\vec{b} - (\vec{b} \cdot \vec{b})\vec{a}\right\}\right]$

$= (2\vec{a} - \vec{b}) \cdot (\vec{b} - 2\vec{a})$

$= -4|\vec{a}|^2 - 4|\vec{b}|^2$

$= -4 - 1 = -5$

20. The value of p and q for which the function

$$f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x} & , x < 0 \\ q & , x = 0 \\ \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}} & , x > 0 \end{cases}$$

is continuous for all x in R, are,

(1) $p = \dfrac{1}{2}, q = \dfrac{3}{2}$ **(2)** $p = \dfrac{1}{2}, q = -\dfrac{3}{2}$

(3) $p = \dfrac{5}{2}, q = \dfrac{3}{2}$ **(4)** $p = -\dfrac{3}{2}, q = \dfrac{1}{2}$

Ans: [4] $q = \lim_{x \to 0^+} f(x)$

$$= \lim_{x \to 0^+} \frac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}}$$

$$= \lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$$

$$= \lim_{x \to 0} \frac{(1+x) - 1}{x} \cdot \frac{1}{\sqrt{1+x} + 1}$$

$$= \frac{1}{2}$$

$$q = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin(p+1)x + \sin x}{x}$$

$$= \lim_{x \to 0^-} \left[\frac{\sin(p+1)x}{(p+1)x} \cdot (p+1) + \frac{\sin x}{x} \right]$$

$$= (p+1) + 1 = p + 2$$

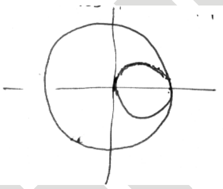
$$\Rightarrow \quad p = q - 2 = \frac{1}{2} - 2 = -\frac{3}{2}$$

21. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if

(1) $|a| = 2c$ **(2)** $2|a| = c$

(3) $|a| = c$ **(4)** $a = 2c$

Ans: [3]



Diameter of $x^2 + y^2 = ax$ = Radius of $x^2 + y^2 = c^2$

22. Let be the purchase value of an equaipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential euqation $\frac{dV(t)}{dt} = -k(T - t)$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is

(1) e^{-kT} **(2)** $T^2 - \frac{1}{k}$

(3) $I - \frac{kT^2}{2}$ **(4)** $I - \frac{k(T-t)^2}{2}$

Ans: [3] $\frac{dV}{dt} = -kT + kt$

$$\int_I^{V(T)} dV = \int_0^T (-kT + kt) dt$$

$$\Rightarrow \quad V(T) - I = \left(-kTt + k\frac{t^2}{2} \right) \Big|_0^T$$

$$= -kT^2 + k\frac{T^2}{2} = -\frac{kT^2}{2}$$

$$\therefore \quad V(T) = I - \frac{kT^2}{2}$$

23. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is

(1) $P(C|D) = \frac{P(D)}{P(C)}$ **(2)** $P(C|D) = P(C)$

(3) $P(C|D) \geq P(C)$ **(4)** $P(C|D) < P(C)$

Ans: [3] $P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)}$ $(\because C \subset D)$

$$\because \quad O \leq P(D) \leq 1$$

$$\Rightarrow \quad P\left(\frac{C}{D}\right) \geq P(C)$$

24. Let A and B be two symmetric matrices of order 3

Statement -1

A (BA) and (AB) A are symmetric matrices

Statement -2

AB is symmetric matrix if matrix multiplication of A with B commutative.

(1) Statement -1 is false, Statement -2 is true

(2) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for statement-1

(3) Statement-1 is true, Statement -2 is true, Statement-2 is not correct explanation for Statement-1

(4) Statement -1 is true, Statement-2 is false

Ans: [3] $A(BA) = (AB)A = ABA$

$$(ABA)^T = A^T B^T A^T = ABA$$

$\therefore$ statement 1 is TRUE

Given $AB = BA$

$$(AB)^T = B^T A^T = BA = AB$$

$\therefore$ AB is symmetric

statement 2 is TRUE but NOT a correct explanationof statement 1.

25. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals

(1) (–1, 1) **(2)** (0, 1)
(3) (1, 1) **(4)** (1, 0)

Ans: **[3]** $A + B\omega = (1+\omega)^7 = \left(-\omega^2\right)^7$

$= -\omega^{14} = -\omega^2 = 1+\omega$

$\Rightarrow \quad A = 1,\ B = 1$

26 **Statement -1**
The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.
Statement -2
The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$

(1) Statement -1 is false, Statement -2 is true
(2) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for statement-1
(3) Statement-1 is true, Statement -2 is true, Statement-2 is not correct explanation for Statement-1
(4) Statement -1 is true, Statement-2 is false

Ans: **[3]** Statement 1 number of way's of distributing identical things into m person when each receive at least one

$= {}^{n-1}C_{m-1}$

hence total ways $= {}^{10-1}C_{4-1} = {}^9C_3$

Statement 2 is correct but not correct explanation of staement 1.

27 The shortest distance between line $y - x = 1$ and curve $x = y^2$ is

(1) $\frac{4}{\sqrt{3}}$ **(2)** $\frac{\sqrt{3}}{4}$

(3) $\frac{3\sqrt{2}}{8}$ **(4)** $\frac{8}{3\sqrt{2}}$

Ans: **[3]** Let a point $P\left(t^2, t\right)$ on the parabola. Distance of the line $y = x + 1$ from P

$= \frac{\left|t^2 - t + 1\right|}{\sqrt{2}} = \frac{t^2 - t + 1}{\sqrt{2}} \qquad \because\ t^2 - t + 1 > 0$

$= \frac{\left(t - \frac{1}{2}\right)^2 + \frac{3}{4}}{\sqrt{2}}$

$\geq \frac{\frac{3}{4}}{\sqrt{2}} \qquad$ (when $t = \frac{1}{2}$)

$= \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$

28. The area of the region enclosed by trhe curves $y = x, x = e, y = (1/x)$ and the positive x-axis is

(1) 5/2 square units
(2) 1/2 square units
(3) 1 square units
(4) 3/21 square units

Ans: **[4]** Area $= \int_0^1 x\,dx + \int_1^e \frac{1}{x}\,dx$

$= \left(\frac{x^2}{2}\right)_0^1 + (\ln x)_1^e = \frac{1}{2} + 1 = \frac{3}{2}$

29. If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2,$ the $y(\ln 2)$ is equal to:

(1) –2 **(2)** 7
(3) 5 **(4)** 13

Ans: **[2]** $\frac{dy}{dx} = y + 3 \qquad \Rightarrow \frac{dy}{y+3} = dx$

$\Rightarrow \ln(y+3) = x + C \qquad \Rightarrow y + 3 = C\cdot e^x$

Now $y(0) = 2 \qquad \Rightarrow 2 + 3 = C \quad \Rightarrow C = 5$

$\therefore \quad y = 5e^x - 3 \qquad \Rightarrow y(\ln 2) = 5e^{\ln 2} - 3 = 7$

30. The vectors $\vec{a}$ and $\vec{b}$ are no perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b}\times\vec{c} = \vec{b}\times\vec{d}$ and $\vec{a}\cdot\vec{d} = 0$. Then the vector $\vec{d}$ is equal to:

(1) $\vec{c} - \left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$ **(2)** $\vec{b} - \left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$

(3) $\vec{c} + \left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$ **(4)** $\vec{b} + \left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$

Ans: **[1]** $\vec{b}\times\vec{c} = \vec{b}\times\vec{d} \qquad \vec{b}\times\left(\vec{c} - \vec{d}\right) = \vec{0}$

$\Rightarrow \quad \vec{c} - \vec{d} = \lambda\vec{b} \qquad \Rightarrow \quad \vec{d} = \vec{c} - \lambda\vec{b}$

Now $\vec{a}\cdot\vec{d} = 0 \qquad \Rightarrow \quad \vec{a}\cdot\vec{c} - \lambda\left(\vec{a}\cdot\vec{b}\right) = 0$

$\Rightarrow \quad \lambda = \frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}} \qquad \therefore \quad \vec{d} = \vec{c} - \left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$

Part -B Chemistry

31. In context of the lanthanoids, which of the following statements is not correct?

(1) Availability of 4f electrons results in the formation of compounds in +4 state for all the members of the series.

(2) There is a gardual decrease in the radii of the members with increasing atomic number in the series.

(3) All the members exhibit +3 oxidation state.

(4) Because of similar properties the separation of lanthanoids is not easy

Sol. **[1]** +3 oxidation state is predominant in lanthanoids.

32. In a face centred cubic lattice, atom A occupies the corner positions and atom B occupies the face centre positions. If one atom of B is missing form one of the face centred points, the formula of the compound is

(1) A_2B_5 **(2)** A_2B

(3) AB_2 **(4)** A_2B_3

Sol. **[1]** A : B

1 : $5\times\frac{1}{2}=\frac{5}{2}$

2 : 5 i.e., A_2B_5

33. The magnetic moment (spin only) of $[NiCl_4]^{4-}$ is

(1) 1.41 BM **(2)** 1.82 BM

(3) 5.46 BM **(4)** 2.82 BM

Sol. **[4]** Outer orbital complex, 2 unpaired electrons

$\mu_{spin}=\sqrt{2(2+2)}=\sqrt{8}=2.82$ B.M.

34. Which of the following facts about the complex $[Cr(NH_3)_6]Cl_3$ is wrong?

(1) The complex gives which precipitate with silver nitrate solution.

(2) The complex involves d^2sp^3 hybridisation and is octahedral in shape.

(3) The complex is paramagnetic

(4) The complex is an outer orbital complex.

Sol. **[4]** It is an inner orbital complex.

35. The rate of a chemical reaction doubles for every $10°C$ rise of temperature. If the temperature is sraised by $50°C$ the rate of the reaction increases by aobut:

(1) 64 times **(2)** 10 times

(3) 24 times **(4)** 32 times

Sol. **[4]** Temperature increased by $50°C$, so change in rate constant will be $(2)^5 \Rightarrow 32$

36. 'a' and 'b' are van der Waals' constants for gases. Chlorine is more easily liquefied than ethane because

(1) a for Cl_2 > a for C_2H_6 but b for Cl_2 < b for C_2H_6

(2) a and b for Cl_2 > a and b for C_2H_6

(3) a and b for Cl_2 < a and b for C_2H_6

(4) a for Cl_2 < a for C_2H_6 but b for Cl_2 > b for C_2H_6

Sol. **[2]** For easy to liquify value of a and b both should be greater.

37. The hybridisation of orbitals of N atom in NO_3^-, NO_2^+ and NH_4^+ are respectively:

(1) sp^2, sp^3, sp **(2)** sp, sp^2, sp^3

(3) sp^2, sp, sp^3 **(4)** sp, sp^3, sp^2

Sol. **[3]** O=N(=O)=O , O=N=O , H—N(H)(H)—H

38. Ethylene glycol is used as an antifreeze in a cold climate. Mass of ethylene glycol which should be added to 4 kg of water to prevent it form freezing at $-6°C$ will be: (K_f foer water $= 1.86$ K kg mol^{-1} and molar mass of ethylene glycol $= 62$ g mol^{-1})

(1) 304.60 g **(2)** 804.32 g

(3) 204.30 g **(4)** 400.00 g

Sol. **[2]** $\Delta T_f = 6 \Rightarrow \Delta T_f = k_f m$

$6 = 1.86\times\frac{W}{62\times 4} \Rightarrow w=\frac{6\times 62\times 4}{1.86}=800$

Any mass which is greater than 800 g will not allow solution to freeze at $-6°C$.

39. The outer electron configuration of Gd (Atomic No : 64) is

(1) $4f^7\,5d^1\,6s^2$ **(2)** $4f^3\,5d^5\,6s^2$

(3) $4f^8\,5d^0\,6s^2$ **(4)** $4f^4\,5d^4\,6s^2$

Sol. **[1]** $[Xe]4f^7 5d^1 6s^2$

40. The structure of IF_7 is

(1) pentagonal bipyramid
(2) square pyramid
(3) trigonal bipyramid
(4) octahedral

Sol. **[1]** sp^3d^3 Hybridization

41. Ozonolysis of an organic compound gives formaldehyde as one of the products. This confirms the presence of:

(1) an acetylenic triple bond
(2) two ethylenic double bonds
(3) a vinyl group
(4) an isopropyl group

Sol. **[3]** Since formaldehyde is formed on ozonolysis, presence of vinyl is indicated.

42. The degree of dissociation (α) of a weak electrolyte, A_xB_y is related to van't Hoff factor (i) by the expression:

(1) $\alpha = \dfrac{x+y+1}{i-1}$ **(2)** $\alpha = \dfrac{i-1}{(x+y-1)}$

(3) $\alpha = \dfrac{i-1}{x+y-1}$ **(4)** $\alpha = \dfrac{x+y-1}{i-1}$

Sol. **[2]** For dissociation

$$A_x B_y \longrightarrow xA^{y+} + yB^{x-}$$

After dissociation $1-\alpha$ $\quad x\alpha \quad y\alpha$

$$\text{Van't Hoff factor } (i) = \frac{\text{no. of molse after dissociation}}{\text{no. of molse before dissociation}}$$

$$i = \frac{1+(x+y-1)\alpha}{1} \Rightarrow i-1 = (x+y-1)\alpha$$

$$\Rightarrow \alpha = \frac{i-1}{x+y-1}$$

43. A gas absorbs a photon of 355 nm and emists at two wavelengths. If one of the emissions is at 680 nm, the other is at:

(1) 518 nm **(2)** 1035 nm
(3) 325 nm **(4)** 743 nm

Sol. **[4]** According to energy conservation

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$\Rightarrow \frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \qquad \Rightarrow \frac{1}{355} = \frac{1}{680} + \frac{1}{\lambda_2}$$

$$\Rightarrow \lambda_2 = 743\text{ nm}$$

44. Identify the compound that exhibits tautomerism.

(1) Phenol **(2)** 2-Butene
(3) Lactic acid **(4)** 2-Pentanone

Sol. **[4]** $CH_3-\overset{\overset{\Large O}{\|}}{C}-CH_2-CH_2-CH_3$ (keto form) $\rightleftharpoons$

$CH_3-\overset{\overset{\Large OH}{|}}{C}=CH-CH_2-CH_3$ (enol form).

45. The entropy change involved int he isothermal reversible expansion of 2 moles of an ideal gas form a volume of 10 dm^3 to a volume of 100 dm^3 at $27°C$ is

(1) $42.3\text{ J mol}^{-1}\text{ K}^{-1}$ **(2)** $38.3\text{ J mol}^{-1}\text{ K}^{-1}$

(3) $35.8\text{ J mol}^{-1}\text{ K}^{-1}$ **(4)** $32.3\text{ J mol}^{-1}\text{ K}^{-1}$

Sol. **[2]** Entropy change for isothermal process

$$\Delta S = nR\ln\frac{V_f}{V_i} \qquad = 2\times\frac{25}{3}\ln\frac{100}{10}$$

$$\Rightarrow 2\times\frac{25}{3}\times 2.303 = 38.3\text{ J mol}^{-1}\text{ K}^{-1}$$

46. Silver Mirror test is given by which one of the following compounds?

(1) Benzophenone **(2)** Acetaldehyde
(3) Acetone **(4)** Formaldehyde

Sol. **[2, 4]** Silver miror test is the characteristic test of aldehydes.

47. Trichloroacetaldehyde was subjected to Cannizszaro's reaction by using NaOH . The mixture of the products contains sodium trichloroacetate and another compound. The other compound is:

(1) Chloroform **(2)** 2, 2, 2-Trichloroethanol
(3) Trichloromethanol **(4)** 2, 2, 2-Trichloropropanol

Sol. **[2]** Cannizaro reaction

$$CCl_3\,CHO \xrightarrow{NaOH} CCl_3COO^-Na^+ + \underset{2,2,2-\text{Trichloroethanol}}{CCl_3CH_2OH}$$

48. The reduction potential of hydrogen half cell will be negative if:

(1) $p(H_2) = 2$ atm and $\left[H^+\right] = 2.0$ M

(2) $p(H_2) = 1$ atm and $\left[H^+\right] = 2.0$ M

(3) $p(H_2) = 1$ atm and $\left[H^+\right] = 1.0$ M

(4) $p(H_2) = 2$ atm and $\left[H^+\right] = 1.0$ M

Sol. **[4]** $2H^+ + 2e^- \longrightarrow H_2$

$$E_{cell} = E^o_{cell} - \frac{0.0591}{2}\log\frac{P_{H_2}}{\left(H^+\right)^2}$$

$$E_{cell} = 0 - \frac{0.0591}{2}\log\frac{P_{H_2}}{\left(H^+\right)^2}$$

for $E_{cell} < 0 \quad \Rightarrow \quad P_{H_2} = 2$ atm, and $\left(H^+\right) = 1.0$ M

49. Phenol is heated with a solution of mixture of KBr and $KBrO_3$. The major product obtained in the above reaction is:

(1) 2, 4, 6-Tribromophenol
(2) 2-Bromophenol
(3) 3-Bromophenol
(4) 4-Bromophenol

Sol. **[1]** $KBrO_3$ and KBr mixture gives Br_2 . Phenol reacts with bromine water to give 2, 4, 6-tribromophenol

50. Among the following the maximum covalent character is shown by the compound:

(1) $MgCl_2$ **(2)** $FeCl_2$
(3) $SnCl_2$ **(4)** $AlCl_3$

Sol. **[4]** Smaller the size of cation more will be the covalent character.

51. Boron cannot form which one of the following anions?

(1) BO_2^- **(2)** BF_6^{3-}
(3) BH_4^- **(4)** $B(OH)_4^-$

Sol. **[2]** Boron cannot expand its octet. Hence it does not form BF_6^{3-}

52. Sodium ethoxide has reacted with ethanoyl chloride. The compound that is produced in the above reaction is

(1) Ethyl ethanoate **(2)** Diethyl ether
(3) 2-Butanone **(4)** Ethyl chloride

Sol. **[1]** $CH_3COCl + C_2H_5ONa \longrightarrow CH_3COOC_2H_5$

53. Which of the following reagents may be used to distinguish between phenol and benzoic acid

(1) Neutral $FeCl_3$ **(2)** Aqueous NaOH
(3) Tollen's reagent **(4)** Molisch reagent

Sol. **[1]** With neutral $FeCl_3$ phenol gives violet colour where as benzoic acid gives buff colored precipitate

54. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm. Some of the CO_2 is converted into CO on the addition of graphite. If the total pressure at equilibrium is 0.8 atm the value of K is

(1) 0.18 atm **(2)** 1.8 atm
(3) 3 atm **(4)** 0.3 atm

Sol. **[2]** $CO_2(g) + C(s) \longrightarrow 2CO(g)$

$0.5 - x \qquad\qquad 2x$

$0.2 \qquad\qquad\qquad 0.6$

$0.5 - x + 2x = 0.8$

$\Rightarrow 0.5 + x = 0.8 \qquad \Rightarrow x = 0.3$

$$\Rightarrow K = \frac{(0.6)^2}{0.2} = 1.8$$

55. The strongest acid amongst the following compounds is

(1) $ClCH_2CH_2CH_2COOH$

(2) CH_3COOH

(3) HCOOH

(4) $CH_3CH_2CH(Cl)CO_2H$

Sol. **[4]** Due to lesser distance from $-COOH$ group electron withdrawing inductive effect of $-Cl$ group is more hence most acidic.

56. Which one of the following orders presents the correct sequence of the increasing basic nature of the given oxides?

(1) $K_2O < Na_2O < Al_2O_3 < MgO$

(2) $Al_2O_3 < MgO < Na_2O < K_2O$

(3) $MgO < K_2O < Al_2O_3 < Na_2O$

(4) $MgO < K_2O < MgO < Al_2O_3$

Sol. **[2]** In a group on moving down basic nature of oxide increases while in a period left to right basic nature decreases.

57. A 5.2 molal aqueous solution of methyl alcohol, CH_3OH is supplied. What is the mole fraction of methyl alcohol in the solution?

(1) 0.050 **(2)** 1.100
(3) 0.190 **(4)** 0.086

Sol. **[4]** 5.2 mol (CH_3OH) in 1000 g water that is 55.55 mols of water.

mol fraction of $CH_3OH = \frac{5.2}{5.2 + 55.55} = 0.086$

58. The presence or absence of hydroxy group on which carbon atom of sugar differentiates RNA and DNA?

(1) 4th **(2)** 1st
(3) 2nd **(4)** 3rd

Sol. **[3]** DNA contain 2-deoxy ribose sugar.

59. Which of the following statement is wrong?

(1) N_2O_4 has two resonance structures.

(2) The stability of hydrides increases form NH_3 to BiH_3 in group 15 of the periodic table.

(3) Nitrogen cannot form $d\pi - p\pi$ bond.

(4) Single N– N bond is weaker than the single $P-P$ bond.

Sol. **[2]** The order of stability of hydrides of 15 group is as follows

$NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$

60. Which of the following statements regarding sulphur is incorrect?

(1) The oxidation state of sulphur is never less than +4 in its compounds.

(2) S_2 molecular is paramagnetic

(3) The vapour at $200°C$ consists mostly of S_8 rings.

(4) At $600°C$ the gas mainly consists of S_2

Sol. **[1]** Sulphur can also show +2, 0 and –2 oxidation states .